UKMT

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 30th APRIL 2015

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

Institute
and Faculty
of Actuaries

**RULES AND GUIDELINES** (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
    No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 8 or below.
    Candidates in Scotland must be in S2 or below.
    Candidates in Northern Ireland must be in School Year 9 or below.

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the
    Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
    When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
    Six marks are awarded for each correct answer to Questions 16-25.
    Each incorrect answer to Questions 16-20 loses 1 mark.
    Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
    sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
    even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
    rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
    and more satisfaction, by doing one question carefully than by guessing lots of answers.
    The UK JMC is about solving interesting problems, not about lucky guessing.

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1. Which of the following calculations gives the largest answer?

   A $1 - 2 + 3 + 4$ B $1 + 2 - 3 + 4$ C $1 + 2 + 3 - 4$ D $1 + 2 - 3 - 4$ E $1 - 2 - 3 + 4$

2. It has just turned 22:22. How many minutes are there until midnight?

   A 178   B 138   C 128   D 108   E 98

3. What is the value of $\dfrac{12\ 345}{1 + 2 + 3 + 4 + 5}$?

   A 1   B 8   C 678   D 823   E 12 359

4. In this partly completed pyramid, each rectangle is to be filled with the sum of the two numbers in the rectangles immediately below it.

   What number should replace x?
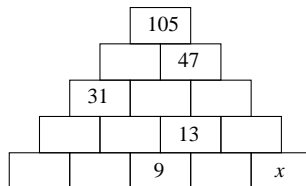
   A 3   B 4   C 5   D 7   E 12

5. The difference between $\dfrac{1}{3}$ of a certain number and $\dfrac{1}{4}$ of the same number is 3. What is that number?

   A 24   B 36   C 48   D 60   E 72

6. What is the value of x in this triangle?

   A 45   B 50   C 55   D 60   E 65
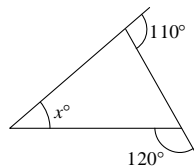
7. The result of the calculation 123 456 789 × 8 is almost the same as 987 654 321 except that two of the digits are in a different order. What is the sum of these two digits?

   A 3   B 7   C 9   D 15   E 17

8. Which of the following has the same remainder when it is divided by 2 as when it is divided by 3?

   A 3   B 5   C 7   D 9   E 11

9. According to a newspaper report, "A 63-year-old man has rowed around the world without leaving his living room." He clocked up 25 048 miles on a rowing machine that he received for his 50th birthday.
   Roughly how many miles per year has he rowed since he was given the machine?

   A 200   B 500   C 1000   D 2000   E 4000

10. In the expression $1\ \square\ 2\ \square\ 3\ \square\ 4$ each $\square$ is to be replaced by either + or ×.
    What is the largest value of all the expressions that can be obtained in this way?

    A 10   B 14   C 15   D 24   E 25

11. What is the smallest prime number that is the sum of three different prime numbers?

   A 11   B 15   C 17   D 19   E 23

12. A fish weighs the total of 2 kg plus a third of its own weight. What is the weight of the fish in kg?

   A $2\frac{1}{3}$ B 3   C 4   D 6   E 8

13. In the figure shown, each line joining two numbers is to be labelled with the sum of the two numbers that are at its end points.
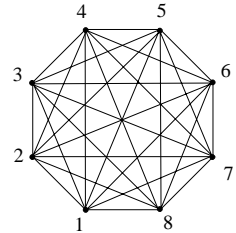
   How many of these labels are multiples of 3?

   A 10   B 9   C 8   D 7   E 6

14. Digits on a calculator are represented by a number of horizontal and vertical illuminated bars. The digits and the bars which represent them are shown in the diagram.

   How many digits are both prime and represented by a prime number of illuminated bars?

   A 0   B 1   C 2   D 3   E 4

15. Which of the following is divisible by all of the integers from 1 to 10 inclusive?

   A 23×34 B 34×45 C 45×56 D 56×67 E 67×78

16. The diagram shows a square inside an equilateral triangle.
   What is the value of $x + y$?

   A 105   B 120   C 135   D 150   E 165
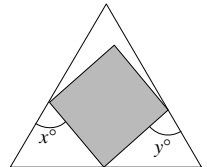
17.
   Knave of Hearts: "I stole the tarts."
   Knave of Clubs: "The Knave of Hearts is lying."
   Knave of Diamonds: "The Knave of Clubs is lying."
   Knave of Spades: "The Knave of Diamonds is lying."

   How many of the four Knaves were telling the truth?

   A 1   B 2   C 3   D 4   E more information needed

18. Each of the fractions $\frac{2637}{18\,459}$ and $\frac{5274}{36\,918}$ uses the digits 1 to 9 exactly once. The first fraction simplifies to $\frac{1}{7}$. What is the simplified form of the second fraction?

   A $\frac{1}{8}$ B $\frac{1}{7}$ C $\frac{5}{34}$ D $\frac{9}{61}$ E $\frac{2}{7}$
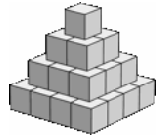
19. One of the following cubes is the smallest cube that can be written as the sum of three positive cubes. Which is it?

    A 27   B 64   C 125   D 216   E 512

20. The diagram shows a pyramid made up of 30 cubes, each measuring $1\,\text{m} \times 1\,\text{m} \times 1\,\text{m}$.

What is the total surface area of the whole pyramid (including its base)?

    A $30\,\text{m}^2$ B $62\,\text{m}^2$ C $72\,\text{m}^2$ D $152\,\text{m}^2$ E $180\,\text{m}^2$

21. Gill is now 27 and has moved into a new flat. She has four pictures to hang in a horizontal row on a wall which is 4800 mm wide. The pictures are identical in size and are 420 mm wide. Gill hangs the first two pictures so that one is on the extreme left of the wall and one is on the extreme right of the wall. She wants to hang the remaining two pictures so that all four pictures are equally spaced. How far should Gill place the centre of each of the two remaining pictures from a vertical line down the centre of the wall?
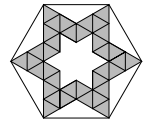
    A 210 mm   B 520 mm   C 730 mm   D 840 mm   E 1040 mm

22. The diagram shows a shaded region inside a regular hexagon. The shaded region is divided into equilateral triangles.

What fraction of the area of the hexagon is shaded?

    A $\dfrac{3}{8}$ B $\dfrac{2}{5}$ C $\dfrac{3}{7}$ D $\dfrac{5}{12}$ E $\dfrac{1}{2}$

23. The diagram shows four shaded glass squares, with areas 1 cm², 4 cm², 9 cm² and 16 cm², placed in the corners of a rectangle. The largest square overlaps two others. The area of the region inside the rectangle but not covered by any square (shown unshaded) is 1.5 cm².

What is the area of the region where squares overlap (shown dark grey)?

    A $2.5\,\text{cm}^2$ B $3\,\text{cm}^2$ C $3.5\,\text{cm}^2$ D $4\,\text{cm}^2$ E $4.5\,\text{cm}^2$
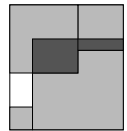
24. A *palindromic number* is a number that reads the same when the order of its digits is reversed. What is the difference between the largest and smallest five-digit palindromic numbers that are both multiples of 45?

    A 9180   B 9090   C 9000   D 8910   E 8190

25. The four straight lines in the diagram are such that $VU = VW$. The sizes of $\angle UXZ$, $\angle VYZ$ and $\angle VZX$ are $x°$, $y°$ and $z°$.

Which of the following equations gives x in terms of y and z?
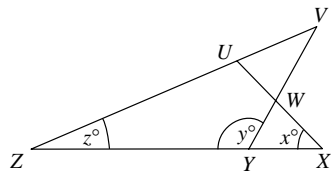
    A $x = y - z$ B $x = 180 - y - z$ C $x = y - \dfrac{z}{2}$

    D $x = y + z - 90$ E $x = \dfrac{y - z}{2}$