UKMT UKMT UKMT

UKMT

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 28th APRIL 2016

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

Institute
and Faculty
of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.

2. Time allowed: **1 hour**.
    No answers, or personal details, may be entered after the allowed hour is over.

3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.

4. Candidates in England and Wales must be in School Year 8 or below.
    Candidates in Scotland must be in S2 or below.
    Candidates in Northern Ireland must be in School Year 9 or below.

5. **Use B or HB non-propelling pencil only**. Mark *at most one* of the options A, B, C, D, E on
    the Answer Sheet for each question. Do not mark more than one option.

6. *Do not expect to finish the whole paper in 1 hour.* Concentrate first on Questions 1-15.
    When you have checked your answers to these, have a go at some of the later questions.

7. Five marks are awarded for each correct answer to Questions 1-15.
    Six marks are awarded for each correct answer to Questions 16-25.
    Each incorrect answer to Questions 16-20 loses 1 mark.
    Each incorrect answer to Questions 21-25 loses 2 marks.

8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the
    sheet except to mark your chosen options**. The machine 'sees' all black pencil markings
    even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of
    rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.

9. The questions on this paper challenge you to **think**, not to guess. You get more marks,
    and more satisfaction, by doing one question carefully than by guessing lots of answers.
    The UK JMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

http://www.ukmt.org.uk

1. Which of the following is closest to zero?

   A $6 + 5 + 4$ B $6 + 5 - 4$ C $6 + 5 \times 4$ D $6 - 5 \times 4$ E $6 \times 5 \div 4$

2. What number is twenty-one less than sixty thousand?

   A 59 979   B 59 981   C 57 900   D 40 001   E 39 000

3. One lap of a standard running track is 400 m.
   How many laps does each athlete run in a 5000 m race?

   A 4   B 5   C 8   D 10   E $12\frac{1}{2}$

4. In January 1859, an eight-year-old boy dropped a newly-hatched eel into a well in Sweden (apparently in order to keep the water free of insects). The eel, named Åle, finally died in August 2014.
   How many years old was Åle when it died?

   A 135   B 145   C 155   D 165   E 175

5. What is the value of $\frac{1}{25} + 0.25$?

   A 0.29   B 0.3   C 0.35   D 0.50   E 0.65

6. Gill is now 28 years old and is a teacher of Mathematics at a school which has 600 pupils. There are 30 more girls than boys at the school.
   How many girls are at Gill's school?

   A 270   B 300   C 315   D 330   E 345

7. A distance of 8 km is approximately 5 miles.
   Which of the following is closest to 1.2 km?

   A 0.75 miles   B 1 mile   C 1.2 miles   D 1.6 miles   E 1.9 miles

8. What is the value of $\dfrac{2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20}{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10}$?

   A 2   B 10   C 20   D 40   E 1024

9. One of the three symbols $+, -, \times$ is inserted somewhere between the digits of 2016 to give a new number. For example, $20 - 16$ gives 4.
   How many of the following four numbers can be obtained in this way?

   $$36 \quad 195 \quad 207 \quad 320$$

   A 0   B 1   C 2   D 3   E 4

10. A square is folded exactly in half and then in half again.
    Which of the following could not be the resulting shape?

    A   B   C   D   E

11. Which of the following statements is false?

   A 12 is a multiple of 2   B 123 is a multiple of 3   C 1234 is a multiple of 4

   D 12 345 is a multiple of 5   E 123 456 is a multiple of 6

12. The musical *Rent* contains a song that starts 'Five hundred and twenty five thousand six hundred minutes'.
   Which of the following is closest to this length of time?

   A a week   B a year   C a decade   D a century   E a millennium

13. The diagram shows five circles placed at the corners of a pentagon.
   The numbers 1, 2, 3, 4, 5 are placed in the circles shown, one in each, so that the numbers in adjacent circles always differ by more than 1.
   What is the sum of the numbers in the two circles adjacent to the circle which contains the number 5?
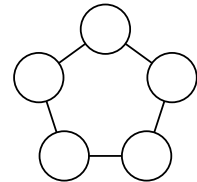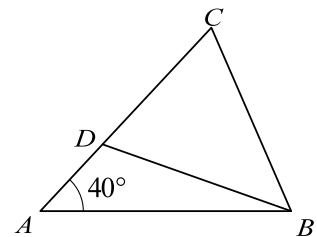

   A 3   B 4   C 5   D 6   E 7

14. In the diagram, $AB = AC$ and D is a point on AC such that $BD = BC$.
   Angle BAC is 40°.
   What is angle ABD?

   A 15°   B 20°   C 25°   D 30°   E 35°

15. How many of these four expressions are perfect squares?

   $1^3 + 2^3$ $1^3 + 2^3 + 3^3$ $1^3 + 2^3 + 3^3 + 4^3$ $1^3 + 2^3 + 3^3 + 4^3 + 5^3$

   A 0   B 1   C 2   D 3   E 4

16. Each of the nine small squares in this grid can be coloured completely black or completely white.
   What is the largest number of squares that can be coloured black so that the design created has rotational symmetry of order 2, but no lines of symmetry?
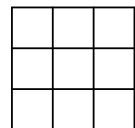
   A 4   B 5   C 6   D 7   E 8

17. In a group of 48 children, the ratio of boys to girls is 3 : 5.
   How many boys must join the group to make the ratio of boys to girls 5 : 3?

   A 48   B 40   C 32   D 24   E 8

18. In the addition sum shown, each letter represents a different non-zero digit.
   What digit does X represent?

$$\begin{array}{r} S\ E\ E \\ +\ S\ E\ E \\ \hline A\ X\ E\ S \end{array}$$

   A 1   B 3   C 5   D 7   E 9

19. Three boxes under my stairs contain apples or pears or both. Each box contains the same number of pieces of fruit. The first box contains all twelve of the apples and one-ninth of the pears.
   How many pieces of fruit are there in each box?

   A 14   B 16   C 18   D 20   E 36

20. A cyclic quadrilateral has all four vertices on the circumference of a circle. Brahmagupta (598−670AD) gave the following formula for the area, A, of a cyclic quadrilateral whose edges have lengths a, b, c, d : $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$, where s is half of the perimeter of the quadrilateral.

What is the area of the cyclic quadrilateral with sides of length 4 cm, 5 cm, 7 cm and 10 cm?
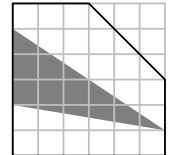
A 6 cm²   B 13 cm²   C 26 cm²   D 30 cm²   E 36 cm²

---
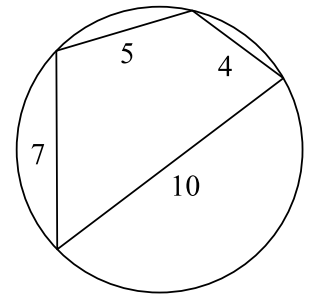
21. The diagram shows a pentagon drawn on a square grid. All vertices of the pentagon and triangle are grid points.
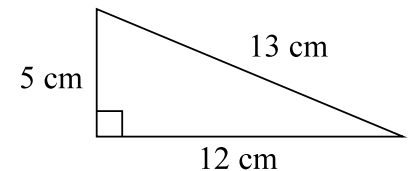
What fraction of the area of the pentagon is shaded?

A $\dfrac{2}{7}$ B $\dfrac{1}{3}$ C $\dfrac{2}{5}$ D $\dfrac{1}{4}$ E $\dfrac{2}{9}$

22. Four copies of the triangle shown are joined together, without gaps or overlaps, to make a parallelogram.

What is the largest possible perimeter of the parallelogram?

A 46 cm   B 52 cm   C 58 cm   D 62 cm   E 76 cm

23. The diagram shows the first few squares of a 'spiral' sequence of squares. All but the first three squares have been numbered. After the first six squares, the sequence is continued by placing the next square alongside three existing squares – the largest existing square and two others.

The three smallest squares have sides of length 1. What is the side length of the 12th square?

A 153   B 123   C 83   D 53   E 13
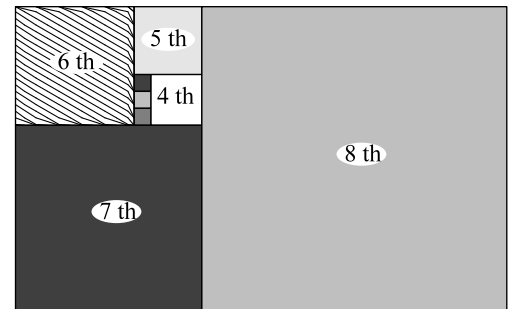
24. Part of a wall is to be decorated with a row of four square tiles. Three different colours of tiles are available and there are at least two tiles of each colour available. Tiles of all three colours must be used.

In how many ways can the row of four tiles be chosen?

A 12   B 18   C 24   D 36   E 48

25. Beatrix places dominoes on a 5 × 5 board, either horizontally or vertically, so that each domino covers two small squares. She stops when she cannot place another domino, as in the example shown in the diagram.

When Beatrix stops, what is the largest possible number of squares that may still be uncovered?

A 4   B 5   C 6   D 7   E 8