UKMT

United Kingdom Mathematics Trust

UK JUNIOR MATHEMATICAL CHALLENGE

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Organised by the **United Kingdom Mathematics Trust from the School of Mathematics, University of Leeds**

http://www.ukmt.org.uk

Institute and Faculty of Actuaries

SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

For reasons of space, these solutions are necessarily brief. There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:

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1. **C** $(222 + 22) \div 2 = 244 \div 2 = 122$.
(*Alternatively*, $(222 + 22) \div 2 = 222 \div 2 + 22 \div 2 = 111 + 11 = 122$.)

2. **D** Before Banbury, 7 people were standing. Therefore the number of people who had no seat after the train left Banbury was equal to $7 - 9 + 28 = 26$.

3. **A** The diagonal of the square is the bisector of a right angle and the interior angle of an equilateral triangle is 60°. Therefore $x = 90 \div 2 + 60 = 45 + 60 = 105$.

4. **E** The perimeter of the regular octagon Q is 8×10 cm $= 80$ cm. So the perimeter of the regular decagon P is 8×80 cm $= 640$ cm. Therefore the length of each edge of P is $(640 \div 10)$ cm $= 64$ cm.

5. **A** The required time is the difference between 06:15 and 08:48. This is a time difference of 2 hours and 33 minutes. So the length of the journey in minutes is $2 \times 60 + 33 = 120 + 33 = 153$.

6. **D** Let the number in the top right corner of the completed magic square be represented by z and let the total of each row, column and main diagonal be T. So, considering the right-hand column of the square, $T = x + y + z$. Also, considering the diagonal from bottom left to top right, $T = 6 + 7 + z$. So $x + y + z = 6 + 7 + z$. Hence $x + y = 6 + 7 = 13$.

7. **B** Note that $20 + 18 = 38$ and $20 \times 18 = 360$. So we need to know the number of integers which are greater than 38 and also less than 360, that is the number of integers from 39 to 359 inclusive. This number is $359 - 39 + 1 = 320 + 1 = 321$.

8. **D** When Gill scored the goal, half of the second quarter of the match remained to be played, plus the third and fourth quarters. So the fraction of the match remaining to be played is equal to $\frac{1}{2} \times \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$.

9. **C** The original cost, in pounds, of building the Flying Scotsman was roughly $\frac{4\,000\,000}{500} = \frac{40\,000}{5} = 8000$.

10. **C** The sum of the five given fractions is $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{18} = \frac{9 + 6 + 3 + 2 + 1}{18}$ $\frac{21}{18} = \frac{7}{6} = 1\frac{1}{6}$. So the fraction which is not used is $\frac{1}{6}$.

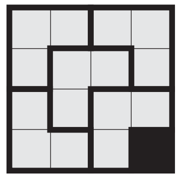
11. **D** The correct answer to the calculation $123\,123\,123\,123 \div 123 = 1\,001\,001\,001$. This has 10 digits.

12. **A** The sum of the interior angles of a triangle equals 180°. So, as $\angle PSR = 110°$, $\angle SPR + \angle PRS = 70°$. Therefore $\angle SPR = \frac{2}{5} \times 70° = 28°$. Now PR bisects $\angle SPQ$, so $\angle QPR = \angle SPR = 28°$. In triangle PQR, we know that $PQ = QR$ so $\angle QRP = \angle QPR = 28°$. Therefore $\angle PQR = 180° - 2 \times 28° = 124°$.

13. **D** Each of the cubes has six faces all of which are exposed. Therefore the four cubes have a total of 24 faces. Each face measures 3 cm by 3 cm and so has an an area of 9 cm^2. Therefore the surface area of the shape is $(24 \times 9)\,\text{cm}^2 = 216\,\text{cm}^2$.

14. **E** Let Billy have b lambs and $3b$ llamas. Let Milly have m llamas and $2m$ lambs. Therefore, as they have 17 animals in total, $4b + 3m = 17$. The only positive integer solution of this equation is $b = 2$, $m = 3$. So the number of llamas is $3b + m = 3 \times 2 + 3 = 6 + 3 = 9$.

15. **D** The diagram shows that it is possible to place five L-shapes on the 4×4 board and, as there is now only one unfilled square, the maximum number of L-shapes which may be placed on the board is five.

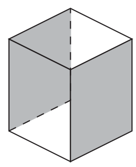


16. **E** The prime factorisation of 15 is 3×5. Therefore for '$p869q$' to be a multiple of 15, the sum of the digits, $p + 8 + 6 + 9 + q$, is a multiple of 3 and $q = 0$ or $q = 5$. When $q = 0$, the sum of the digits is $23 + p$, which is a multiple of 3 when $p = 1$, 4 or 7. When $q = 5$, the sum of the digits is $28 + p$, which is a multiple of 3 when $p = 2$, 5 or 8. So the possible pairs (p, q) are (1, 0), (4, 0), (7, 0), (2, 5), (5, 5) (8, 5). There are six in all.

17. **B** The height of the smaller rectangle is $(13 \div 4)$ cm $= \frac{13}{4}$ cm. Therefore the height of the larger rectangle is $\left(3 + \frac{13}{4}\right)$ cm $= \frac{25}{4}$ cm. So, considering the area of the larger rectangle, $x \times \frac{25}{4} = 25$. Therefore $x = 25 \div \frac{25}{4} = 25 \times \frac{4}{25} = 4$.

18. **B** The first digits of the two numbers will need to be as close as possible to each other. Since they cannot be equal, they will have to differ by 1; say they are n and $n + 1$. The difference between the two numbers will then be minimised by making the four digits after $n + 1$ as small as possible and the four digits after n as large as possible. The smallest four-digit number available is 0123 and the largest is 9876. So we need to make $n = 4$ and then the required diference is $50123 - 49876 = 247$.

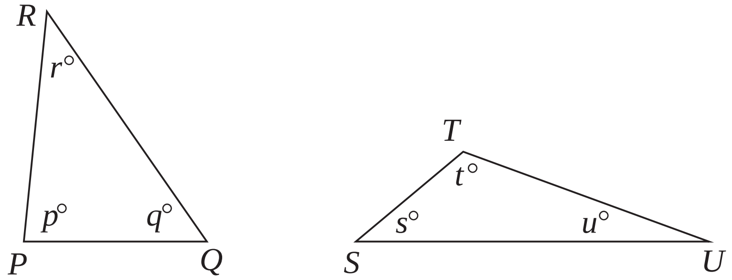
19. **C** All five options contain four squares. When folded, these four squares form the unshaded faces in the diagram shown. So we need to work out in which of the options the two pairs of isosceles right angles fold together to make the two shaded faces. This happens only in option C.



20. **B** Two matching pairs of socks could be obtained by choosing four socks, but this is not certain. For instance, two of one colour, one of a second colour and one of a third colour could be drawn. Combinations of five chosen socks would give two matching pairs unless three socks of one colour, one of a second colour and one of a third colour were chosen. When this is the case, drawing a sixth sock would guarantee that there would be two matching pairs as there would now be either four socks of one colour plus two other socks or three socks of one colour, two of a second colour plus one of a third colour.

21. **D** There are already twelve vowels in the box. So the correct answer corresponds to the number which equals $12 + n$, where n is the number of vowels in the spelling of the number. The only one of the options for which this is true is 'fifteen', which has three vowels and $15 = 12 + 3$.

22. B



In the diagram, the sizes in degrees of the angles in triangles PQR and SUT are denoted by $p°$, $q°$, $r°$, $s°$, $t°$, $u°$. Therefore $p = 2s$; $r = 2u$; $q = \frac{1}{5}t$. Also, as the sum of the interior angles in a triangle is 180°, $p + q + r = 180$… (1) and $s + u + t = 180$… (2).

Substituting for p, q, r in (1) gives $2s + \frac{1}{5}t + 2u = 180$… (3). Equation (2) × 2 gives $2s + 2t + 2u = 360$… (4). Subtracting (3) from (4) gives, $2t - \frac{1}{5}t = 180$. So $\dfrac{9t}{5} = 180$, that is $t = 100$.

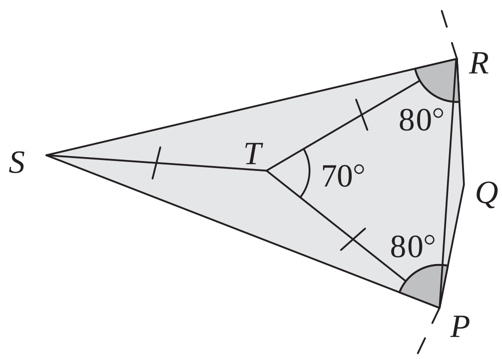
23. A The number in each square after the fourth from the left is the sum of the numbers in the four squares to its left.

2	p	0	q	1	r	8	s

Therefore, $s = q + 1 + r + 8$… (1) and $8 = 0 + q + 1 + r$… (2).
From (2), $q + r = 7$. So, from (1), $s = 7 + 1 + 8 = 16$. So the number which Ali will write in the final square is 16.
{*It is left an exercise for the reader to calculate the numbers in the other three blank squares.*}

24. B Let the perimeter of one of the rectangles be p cm and let the length of one of its shorter sides be x cm. Before the rectangles in figure P are joined together, their total perimeter is $2p$ cm. However, when they are joined together, a length equal to the lengths of two of the shorter sides is 'lost' at the join. So the perimeter of shape P, in cm, equals $2p - 2x$. So $2p - 2x = 58$… (1). Similarly, shape Q consists of three of the rectangles, but there are two joins. So a length equal to the lengths of four of the shorter sides is 'lost' at the joins. Therefore, the perimeter of shape Q, in cm, equals $3p - 4x$. So $3p - 4x = 85$… (2). Multiplying equation (1) by 2 and then subtracting equation (2) gives $4p - 4x - (3p - 4x) = 2 \times 58 - 85$. So $p = 31$. Therefore the perimeter of one of the rectangles is 31 cm.

25. D As PQ and QR are both sides of a regular polygon, they are equal in length. So $\angle PRQ = \angle RPQ$ and hence in triangle PRS, $\angle PRS = \angle RPS$. Therefore triangle PRS is isosceles with $RS = PS$.



Hence triangles RST and PST are congruent (SSS). So $\angle PTS = \angle RTS = \frac{1}{2} \times (360° - 70°) = 145°$. Therefore $\angle TRS = \angle RST = \frac{1}{2} \times (180° - 145°) = 17\frac{1}{2}°$.
So $\angle RSP = 35°$. The sum of the angles in the quadrilateral $SPQR$ is 360° and so $\angle PQR = 360° - 80° - 80° - 35° = 165°$. Therefore the exterior angle of the regular polygon is $180° - 165° = 15°$. So $n = \dfrac{360}{15} = 24$.