United Kingdom
Mathematics Trust

Junior Mathematical Challenge

Solutions 2022

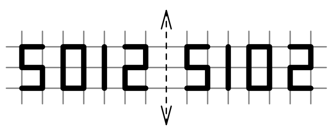
For reasons of space, these solutions are necessarily brief.

There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:

www.ukmt.org.uk

1. E The values of the expressions are A 42; B 204; C 404; D 0; E 440.
So 20×22 has the greatest value.

2. A The diagram shows that 5012 is reflected onto 5102.



3. D Let the number first thought of be x. Adding five gives $x + 5$; multiplying by two then gives $2x + 10$; adding ten now gives $2x + 20$; dividing by two leaves us with $x + 10$, then subtracting the original number gives 10. Finally, adding three means that the final result is 13, whichever number is first thought of.

4. D The value of $0.6 + \frac{2}{5}$ is $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$.

5. C The first and third fractions take integer values: $\frac{1}{1} = 1$; $\frac{111}{1+1+1} = \frac{111}{3} = 37$.
Now consider the other three fractions: 11 is odd, so is not divisible by 2; 1111 is also odd and hence is not divisible by 4 and 11 111 does not have a unit digit of 0 or 5 and therefore is not divisible by 5. So exactly two of the given fractions take integer values.

6. B The interior angles of a square and of an equilateral triangle are 90° and 60° respectively. Therefore, as the angles at a point sum to 360°, $\angle QUP = (360 - 90 - 2 \times 60)° = 150°$.
As square $RSTU$ has sides in common with both of the equilateral triangles, PUT and QRU, the side-lengths of these triangles are equal.
Therefore $QU = UP$, so triangle QUP is isosceles with $\angle QPU = \angle PQU = (180-150)° \div 2 = 15°$.

7. D The weight of kiwi fruit which contains approximately the same amount of vitamin C as 1 kg of oranges is $(1000 \div 2\frac{1}{2})$ g $=(1000 \times \frac{2}{5})$ g $= 400$ g.

8. E Note that $100 \div 7 = 14$ remainder 2, so a period of 100 days is equal to 14 weeks and 2 days.
Therefore, in 100 days' time it will be Saturday.

9. **B** Let the side length of the large square be 9. Then the possible sizes of squares are 9, 6, 3, 2 and 1. The numbers of each size are, respectively, 1, 4, 9, 4 and 9. So the total number of squares in the diagram is $1 + 4 + 9 + 4 + 9 = 27$.

10. **D** Let the number be x. Then $\left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8}\right)x = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$. Therefore $\frac{x}{64} = \frac{7}{8}$. So $x = \frac{64 \times 7}{8} = 56$.

11. **E** The sum of the ten numbers is 55. For the numbers in the two groups to sum to multiples of 4, the total of the nine remaining numbers must itself be a multiple of 4. To achieve this, either 3 or 7 could be left out. If 7 were removed then the remaining numbers total 48. These could then be placed in the required two groups in a number of ways, for example 1, 2, 5 (total 8) and 3, 4, 6, 8, 9, 10 (total 40). Hence the largest number which could be left out is 7.

12. **B** From the information given, it may be deduced that the weight of one quarter of the paint is $(5.8 - 3.1)$ kg $= 2.7$ kg. So the weight of the empty paint pot is $(3.1 - 2.7)$ kg $= 0.4$ kg.
Hence the weight, in kg, of the full pot of paint is $0.4 + 4 \times 2.7 = 0.4 + 10.8 = 11.2$.

13. **D** The inner of the two shaded areas is the difference in area between a square of side 2 cm and a square of side 1 cm. The outer of the two shaded areas is the difference in area between a square of side 4 cm and a square of side 3 cm.
So the total shaded area, in cm^2, is $2^2 - 1^2 + 4^2 - 3^2 = 4 - 1 + 16 - 9 = 10$.
The area of the outer square is $25\,\text{cm}^2$.
Therefore the percentage of the area of the outer square which is shaded is $\frac{10}{25} \times 100\% = 40\%$.

14. **B** Since 13 is directly opposite 35 and the children are evenly spread, there are the same number of children between 13 and 35 going clockwise or anticlockwise. Between 13 and 35, excluding 13 and 35, there are 21 numbers. So between them, in each direction there are 21 children.
Hence the total number of children is $2 \times 21 + 2 = 44$.

15. **E** The value of $2 \div (4 \div (6 \div (8 \div 10)))$ is $2 \div (4 \div (6 \times \frac{10}{8})) = 2 \div (4 \div \frac{15}{2}) = 2 \div (4 \times \frac{2}{15}) = 2 \div \frac{8}{15}$ $= 2 \times \frac{15}{8} = \frac{15}{4}$.

16. **C** The perimeter of the polygon $PQRSTUV$ is equal to the combined perimeters of equilateral triangles PQW and STU minus the perimeter of the overlapping area, namely equilateral triangle RWV. Therefore the required perimeter, in cm, is $3 \times 5 + 3 \times 8 - 3 \times 2 = 3 \times 11 = 33$.

17. **C** After each round of the game, the total number of counters held by the two players increased by two. As they started with a total of 20 counters and finished with a total of 56 counters, the number of rounds played was $(56 - 20) \div 2 = 18$.
Let the number of rounds won by Amrita be n. Then she lost $18 - n$ rounds.
Therefore $10 + (3 \times n) - 1 \times (18 - n) = 40$. Hence $10 + 3n - 18 + n = 40$, that is $4n = 48$.
So Amrita won 12 rounds of the game.

18. **E** As the figure is a parallelogram, the sum of the two angles on the left of the figure is 180°.
So $3x - 40 + 2x - 30 = 180$. Hence $5x = 250$ and therefore $x = 50$. Diagonally opposite angles in a parallelogram are equal. So $4y - 50 = 2x - 30 = 70$. Hence $4y = 120$ and therefore $y = 30$.

19. **B** Let the number of apples and pears at the start of the day be $3n$ and n respectively.
As I had twice as many pears as apples after eating five apples but no pears, $n = 2(3n - 5)$.
Hence $5n = 10$, so $n = 2$.
Therefore the number of pieces of fruit at the start of the day was $4n = 4 \times 2 = 8$.

20. **B** First note that exactly one of Pam and Quentin is telling the truth. Though it is not possible to tell who it is, it means that none of Roger, Susan and Terry is telling the truth.
So just one of the five students is telling the truth.

21. **C** The sum of the numbers in List S is 54 and the corresponding number for List T is 48. Therefore, for the sum of the numbers in List S to equal the sum of the numbers in List T it is necessary that the number which Jenny moves from List S to List T is bigger by 3 than the number which moves from List T to List S. She may achieve this in three ways: by exchanging the 5 in List S with the 2 in list T; by exchanging 8 and 5 or by exchanging 13 and 10.

22. **E** When the pyramid is removed from the cube, the solid loses the edges TU, UQ and UV. However, it also gains the three edges of triangle TQV. So the remaining solid has the same number of edges as the original cube, that is 12.

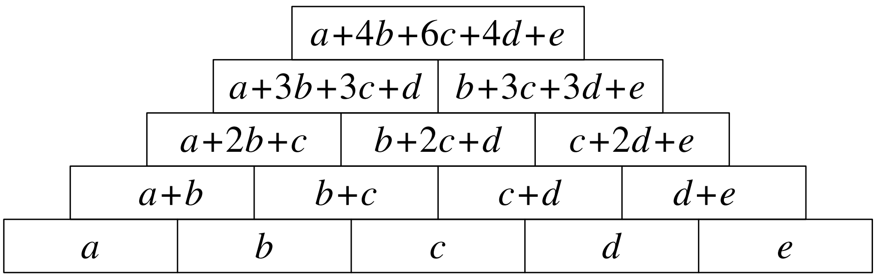
23. **D** Let the original price of the ticket be £P.

Then $P \times 1.05 \times 0.8 = P - 4$. So $P \times 0.84 = P - 4$. Hence $P = \frac{4}{0.16} = \frac{400}{16} = 25$.

Therefore the original cost of the ticket was £25.

24. **A** Let p, y, r and w be the numbers of purple, yellow, red and white flowers, respectively.
So $p : y = 1 : 2$, and $y : r = 3 : 4$ and $r : w = 5 : 6$. Now $1 : 2 = 3 : 6$ and $3 : 4 = 6 : 8$.
Hence $p : y : r = 3 : 6 : 8$. Similarly, $3 : 6 : 8 = 15 : 30 : 40$ and $5 : 6 = 40 : 48$.
So $p : y : r : w = 15 : 30 : 40 : 48$. Note that 1 is the only common factor of these numbers and their total is 133. Hence the total number of flowers must be a multiple of 133 and, being less than 150, it must be 133.

25. **C** Let the numbers on the bottom row of the number pyramid be a, b, c, d, e.
The diagram shows the contents of the other cells in terms of those variables.



So $a + b + c + d + e = 17$ ... [1]; $b + 2c + d = 16$ ... [2]; and $a + 4b + 6c + 4d + e = 61$ ... [3].
[3] - [1] gives $3b + 5c + 3d = 44$ ... [4]; [2] $\times$ 3 gives $3b + 6c + 3d = 48$ ... [5].
Finally, [5] - [4] gives $c = 4$. So the central number of the bottom row is 4.