UK Junior Mathematical Olympiad 2016

Organised by The United Kingdom Mathematics Trust

Tuesday 14th June 2016

RULES AND GUIDELINES :
READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. **Write in blue or black pen or pencil.**
   For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.
   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.
   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.
   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 30 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you are not able to do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

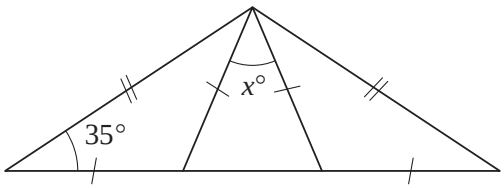
Try to complete Section A within 30 minutes or so. Only answers are required.

A1. Roger picks two consecutive integers, one of which ends in a 5. He multiplies the integers together and then squares the result.

What are the last two digits of his answer?

A2. Three isosceles triangles are put together to create a larger isosceles triangle, as shown.

What is the value of x?

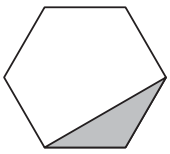


A3. The first term of a sequence is 0. Each term of the sequence after the first term is equal to $10p + 1$, where p is the previous term.

What is the sum of the first ten terms?

A4. The diagram shows a regular hexagon with area 48 m^2.

What is the area of the shaded triangle?



A5. Linda has a very thin sheet of paper measuring 20 cm by 30 cm. She repeatedly folds her paper in half by folding along the shorter line of symmetry. She finishes when she has a rectangle with area 75 cm^2.

What is the perimeter of her final rectangle?

A6. The points A, B, C, D and E lie in that order along a straight line so that $AB : BC = 1 : 2$, $BC : CD = 1 : 3$ and $CD : DE = 1 : 4$. What is $AB : BE$?

A7. A certain positive integer has exactly eight factors. Two of these factors are 15 and 21.

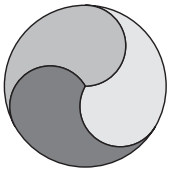
What is the sum of all eight factors?

A8. Julie and her daughters Megan and Zoey have the same birthday. Today, Julie is 32, Megan is 4 and Zoey is 1.

How old will Julie be when her age is the sum of the ages of Megan and Zoey?

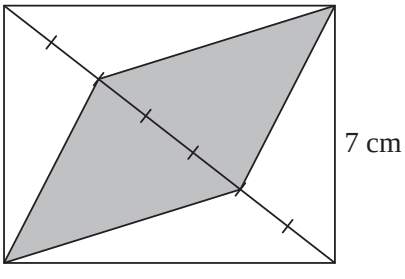
A9. A circle of radius 18 cm is divided into three identical regions by the three semicircles, as shown.

What is the length of the perimeter of one of these regions?



A10. The diagram shows a rectangle with length 9 cm and width 7 cm. One of the diagonals of the rectangle has been divided into seven equal parts.

What is the area of the shaded region?



Section B

Your solutions to Section B will have a major effect on your JMO result. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1. In a certain triangle, the size of each of the angles is a whole number of degrees. Also, one angle is 30° larger than the average of the other two angles.

What is the largest possible size of an angle in this triangle?

B2. The points A, B and C are the centres of three circles. Each circle touches the other two circles, and each centre lies outside the other two circles. The sides of the triangle ABC have lengths 13 cm, 16 cm and 20 cm.

What are the radii of the three circles?

B3. A large cube consists of a number of identical small cubes. The number of small cubes that touch four other small cubes face-to-face is 168.

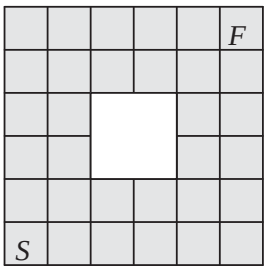
How many small cubes make up the large cube?

B4. In the trapezium $ABCD$, the lines AB and CD are parallel. Also $AB = 2DC$ and $DA = CB$. The line DC is extended (beyond C) to the point E so that $EC = CB = BE$. The line DA is extended (beyond A) to the point F so that $AF = BA$.

Prove that $\angle FBC = 90°$.

B5. The board shown has 32 cells, one of which is labelled S and another F. The shortest path starting at S and finishing at F involves exactly nine other cells and ten moves, where each move goes from cell to cell 'horizontally' or 'vertically' across an edge.

How many paths of this length are there from S to F?



B6. For which values of the positive integer n is it possible to divide the first $3n$ positive integers into three groups each of which has the same sum?