United Kingdom
Mathematics Trust

Junior Mathematical Olympiad

Tuesday 11 June 2019

Organised by the United Kingdom Mathematics Trust

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England & Wales: Year 8 or below
Scotland: S2 or below
Northern Ireland: Year 9 or below

These problems are meant to be challenging.

Do not hurry. Spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you do most of Section A and hand in full solutions to two or more questions in Section B.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. Write your solutions neatly on A4 paper using blue or black pen. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. Only answers are required to the questions in Section A.
8. For questions in Section B, you should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Junior Mathematical Olympiad should be sent to:

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Section A

Try to complete Section A within 30 minutes or so. Only answers are required.

A1. What is the time 1500 seconds after 14:35?

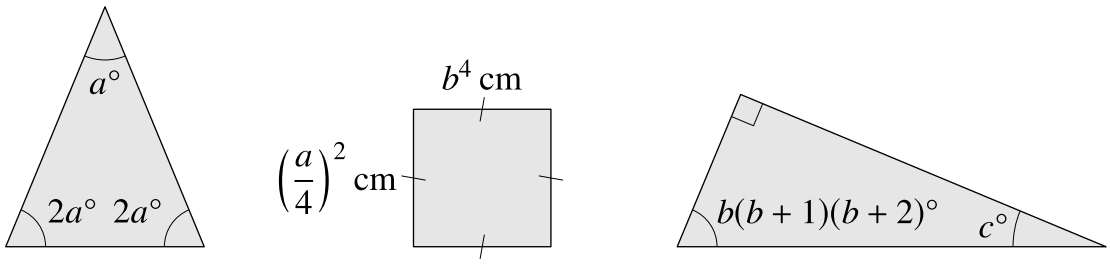
A2. Six standard, fair dice are rolled once. The total of the scores rolled is 32.

What is the smallest possible score that could have appeared on any of the dice?

A3. A satellite orbits the Earth once every 7 hours.

How many orbits of the Earth does the satellite make in one week?

A4.



What is the value of c?

A5. Dani wrote the integers from 1 to N. She used the digit 1 fifteen times. She used the digit 2 fourteen times.

What is N?

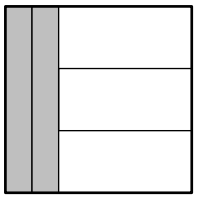
A6. How many fractions between $\frac{1}{6}$ and $\frac{1}{3}$ inclusive can be written with a denominator of 15?

A7. Two 2-digit multiples of 7 have a product of 7007.

What is their sum?

A8. The diagram shows a square made from five rectangles. Each of these rectangles has the same *perimeter*.

What is the ratio of the area of a shaded rectangle to the area of an unshaded rectangle?



A9. The number 3600 can be written as $2^a \times 3^b \times 4^c \times 5^d$, where a, b, c and d are all positive integers. It is given that $a + b + c + d = 7$.

What is the value of c?

A10. Three positive integers add to 93 and have a product of 3375. The integers are in the ratio $1 : k : k^2$.

What are the three integers?

Section B

Your solutions to Section B will have a major effect on your result.

Concentrate firstly on one or two Section B questions and then write out *full solutions* (not just brief 'answers'), including mathematical reasons as to why your method is correct.

You will have done well if you hand in full solutions to two or more Section B questions.

Do *not* hand in rough work.

B1. In this word-sum, each letter stands for one of the digits 0–9, and stands for the same digit each time it appears. Different letters stand for different digits. No number starts with 0.

$$\begin{array}{r} J\ M\ O \\ J\ M\ O \\ +\ J\ M\ O \\ \hline I\ M\ O \end{array}$$

Find all the possible solutions of the word-sum shown here.

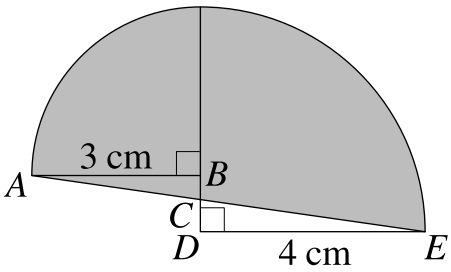
B2. The product $8000 \times K$ is a square, where K is a positive integer.

What is the smallest possible value of K?

B3. It takes one minute for a train travelling at constant speed to pass completely through a tunnel that is 120 metres long. The same train, travelling at the same constant speed, takes 20 seconds from the instant its front enters the tunnel to it being completely inside the tunnel.

How long is the train?

B4. The diagram alongside shows two quarter-circles and two triangles, ABC and CDE. One quarter-circle has radius AB, where $AB = 3$ cm. The other quarter-circle has radius DE, where $DE = 4$ cm.



The area enclosed by the line AE and the arcs of the two quarter-circles is shaded.

What is the total shaded area, in cm^2?

B5. My 24-hour digital clock displays hours and minutes only.

How many displayed times in a 24-hour period contain at least one occurrence of the digit 5?

B6. An equilateral triangle is divided into smaller equilateral triangles.

The diagram on the left shows that it is possible to divide it into 4 equilateral triangles. The diagram on the right shows that it is possible to divide it into 13 equilateral triangles.

What are the integer values of n, where $n > 1$, for which it is possible to divide the triangle into n smaller equilateral triangles?